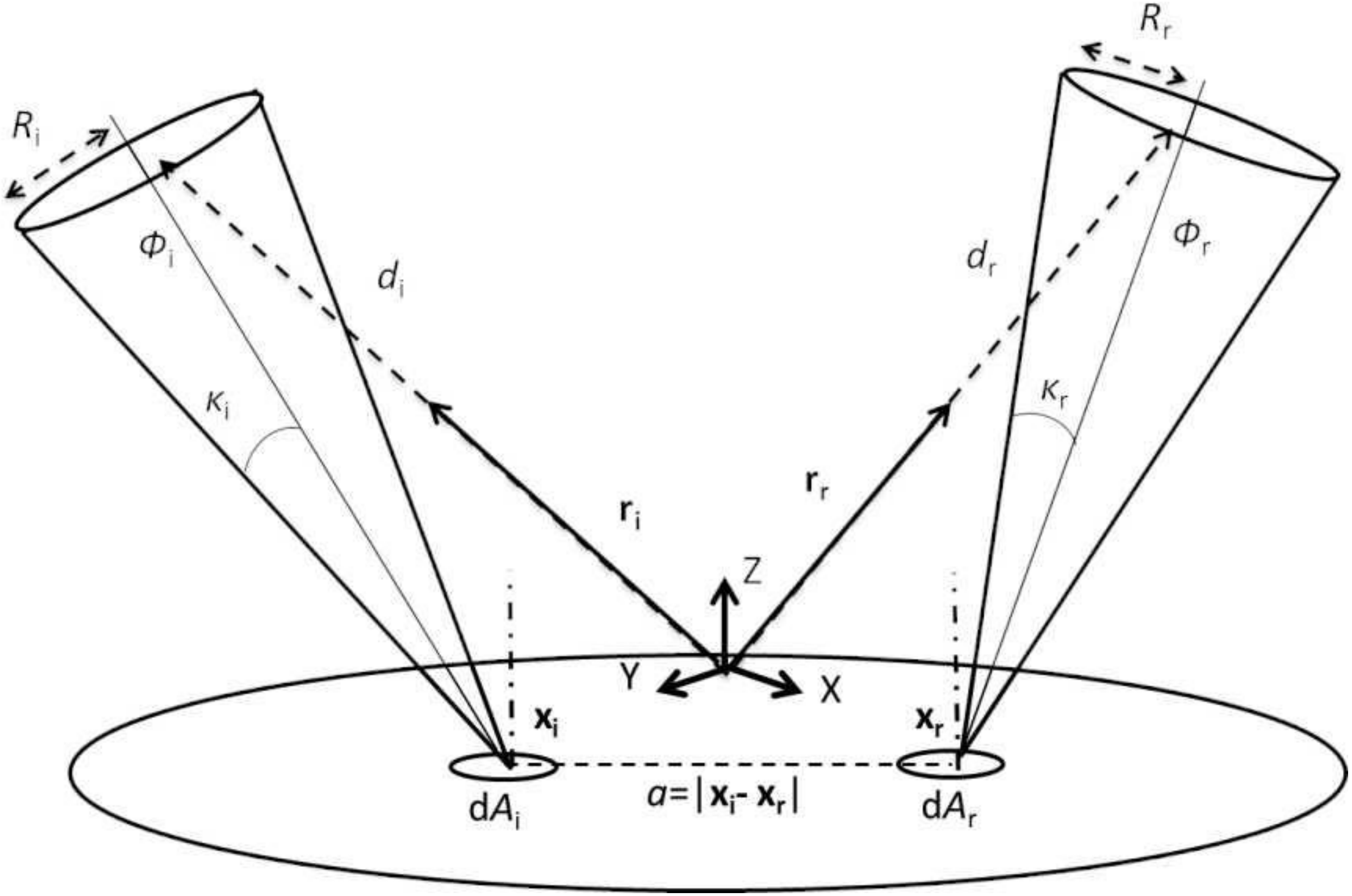
Effect of the size of the irradiation area in BTDF measurements

Alejandro FERRERO
Instituto de Óptica, Consejo Superior de Investigaciones Científicas (CSIC), Spain
alejandro.ferrero@csic.es

Content

- Previous definitions
- Radiance measurements for BTDF
- Irradiance measurements for BTDF
- Conclusions

Previous definitions



Previous definitions

Nicodemus F E, Richmond J C and Hsia J J 1977
Geometrical considerations and nomenclature for reflectance (Natl. Bur. Stand. Monogr. 160).

$$\mathrm{d}L_{\mathrm{r}} = S\mathrm{d}\Phi_{\mathrm{i}} \quad \textbf{(1)}$$

where the basic proportionality function S is the scattering function, which is a property of the reflecting surface.

When only angular and spatial dependence is considered:

$$S = S(\theta_{\mathrm{i}}, \phi_{\mathrm{i}}, x_{\mathrm{i}}, y_{\mathrm{i}}; \theta_{\mathrm{r}}, \phi_{\mathrm{r}}, x_{\mathrm{r}}, y_{\mathrm{r}}) \quad \textbf{(2)}$$

then it is named Bidirectional Scattering–Surface Reflectance Distribution Function or BSSRDF.

BSSRDF

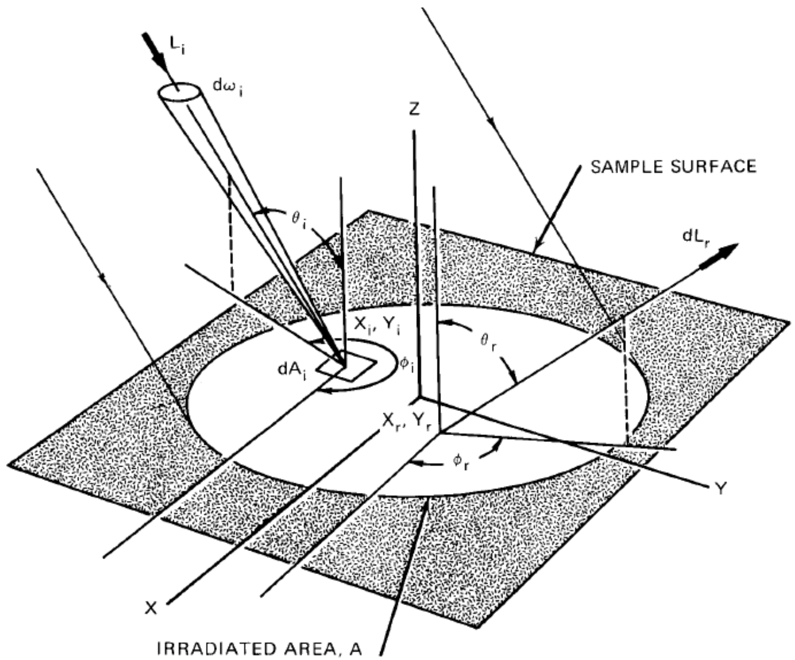
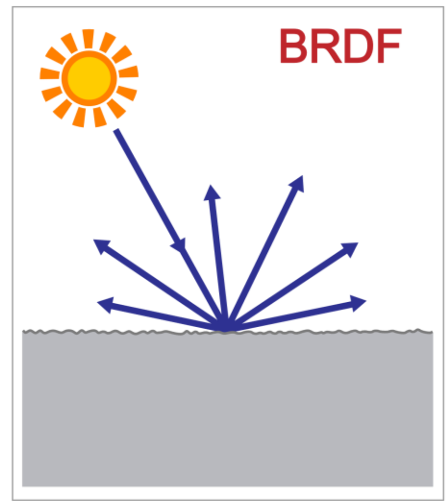
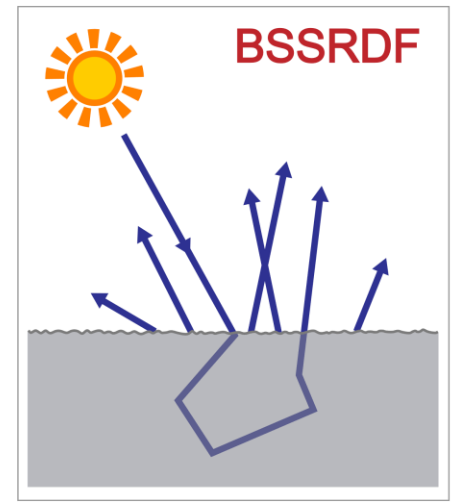


FIGURE 1. *Geometry of incident and reflected beams (for general cases where sub-surface scattering is involved).*

Nicodemus F E, Richmond J C and Hsia J J 1977 *Geometrical considerations and nomenclature for reflectance (Natl. Bur. Stand. Monogr. 160).*



Bidirectional Reflectance distribution function



Bidirectional Scattering –Surface Reflectance distribution function

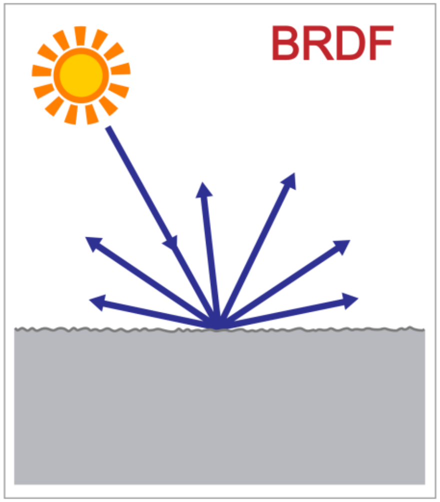
BRDF vs. BSSRDF

$$f_r(\theta_i, \phi_i; \theta_r, \phi_r) = \frac{dL_r}{dE_i} = \frac{dL_r}{L_i \cos\theta_i d\omega_i} \sim \frac{L_r}{L_i \cos\theta_i \omega_i} = \frac{\Phi_r}{\Phi_i \cos\theta_r \omega_r}$$

$$S(\theta_i, \phi_i, x_i, y_i; \theta_r, \phi_r, x_r, y_r) = \frac{dL_r}{d\Phi_i} = \frac{\partial^2 L_r}{L_i \cos\theta_i \, \partial\omega_i \partial A_i} \sim \frac{L_r}{L_i \cos\theta_i \, \omega_i \mathrm{d}A_i} = \frac{\Phi_r}{\Phi_i \cos\theta_r \, \omega_r \mathrm{d}A_i}$$

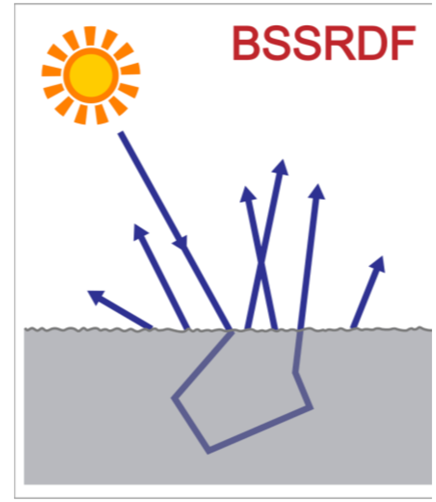
$$f_r(\theta_i, \phi_i; \theta_r, \phi_r) = \int_{A_i} S(\theta_i, \phi_i, x_i, y_i; \theta_r, \phi_r, x_r, y_r) \, \mathrm{d}A_i$$

BRDF vs. BSSRDF



$$L_r = f_r \times E_i$$

L_r is proportional to $\mathbf{E_i}$



$$L_r = S \times \Phi_i$$

L_r is proportional to $\boldsymbol{\Phi}_i$

BUT

$$\Phi_i = E_i \times A_i$$

THEN

$$L_r = S \times E_i \times A_i$$

Radiance at surface is constant for constant irradiance

Radiance due to volume is constant for constant incident radiant flux

A large enough area

Bidirectional reflectance distribution function (BRDF)

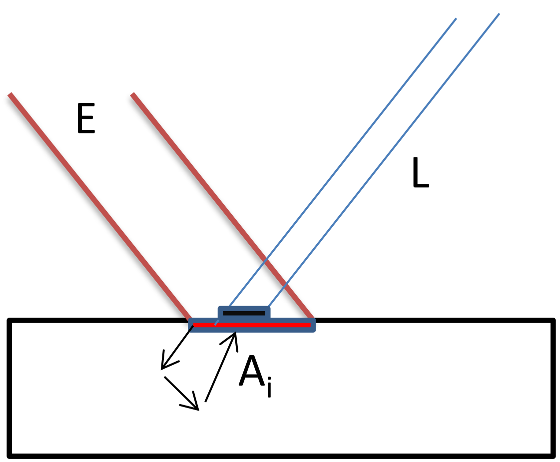
$$f_\mathrm{r}(\mathbf{r}_\mathrm{i};\mathbf{r}_\mathrm{r}) = \left.\frac{\mathrm{d}L_\mathrm{r}(\mathbf{r}_\mathrm{i};\mathbf{r}_\mathrm{r})}{\mathrm{d}E(\mathbf{r}_\mathrm{i})}\right|_{A_\mathrm{i}\text{ large enough}}$$

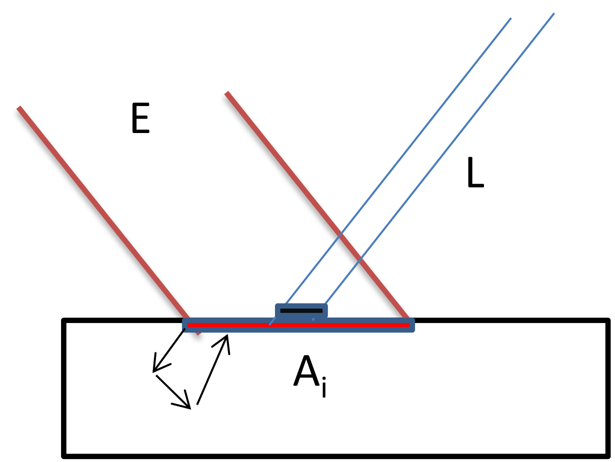
F. E. Nicodemus, J. C. Richmond, J. J. Hsia, I. W. Ginsberg, and T. Limperis, "Geometrical considerations and nomenclature for reflectance," Natl. Bur. Stand. Monogr. ***160****, 0 (1977).*

A large enough area

Bidirectional reflectance distribution function (BRDF)

Why a large enough irradiation area?





<u>No large enough</u>

Light transmitted at the edges can be collected

<u>Large enough:</u>

Light transmitted at the edges is not collected

A large enough area

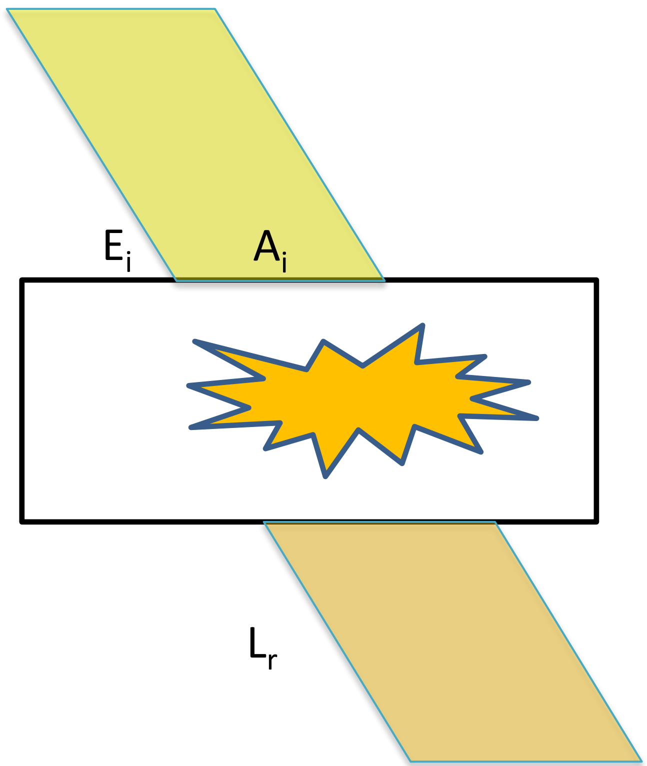
Bidirectional transmittance distribution function (BTDF)

It can be defined similarly as the **Bidirectional Reflectance Distribution Function** (BRDF).

This function describes the bidirectional transmittance for infinitesimal solid angles and for any irradiation ($\mathbf{r}_\mathrm{i}$) and collection ($\mathbf{r}_\mathrm{t}$) directions. The BTDF should allow transmittance to be calculated for any condition via integration.

$$f_\mathrm{t}(\mathbf{r}_\mathrm{i}; \mathbf{r}_\mathrm{t}) = \left.\frac{\mathrm{d}L_t}{\mathrm{d}E_\mathrm{i}}\right|_{A_i\ large\ enough}$$

BTDF in translucent materials



$L_r = L_{r,V}$

Supposedly:

$BTDF = L_r/E_i = L_{r,V}/E_i$

But by definiton:

$L_{r,V} = S \times \Phi_i = S \times E_i \times A_i$

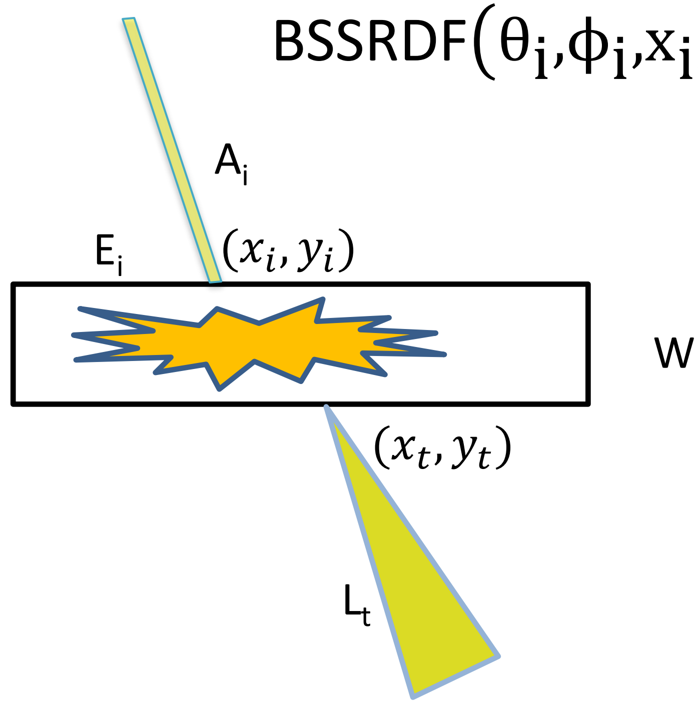
$BTDF = L_r/E_i = S \times A_i$

Radiance measurements for BTDF

Case 1

$$\mathrm{BSSRDF}(\theta_i,\phi_i,x_i,y_i;\theta_t,\phi_t,x_t,y_t) = \frac{dL_t}{d\Phi_i}$$

$W \gg \sqrt{A_i}$



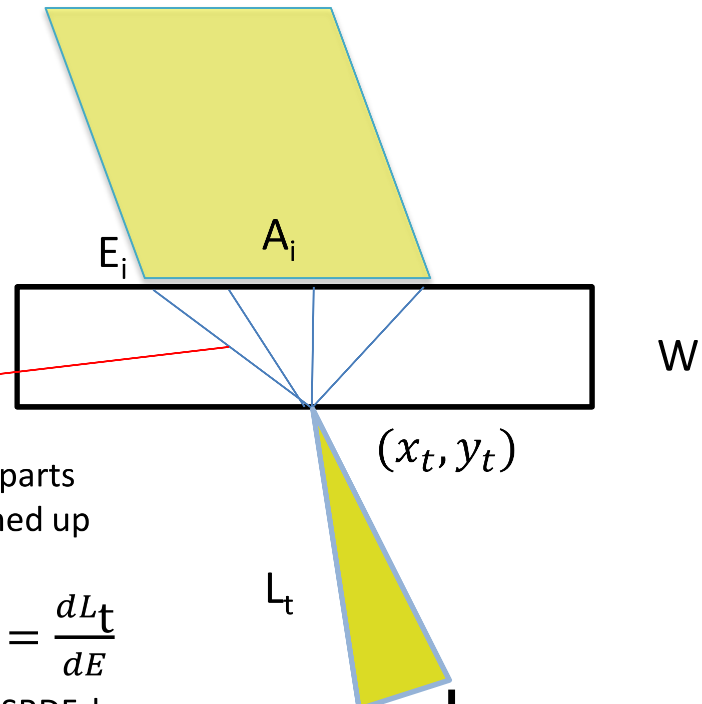
$$\left| 1 - \frac{\mathrm{BSSRDF}(\theta_i, \phi_i, 0{,}0; \theta_t, \phi_t, x_t, y_t)}{\mathrm{BSSRDF}(\theta_i, \phi_i, \sqrt{A_i}, \sqrt{A_i}; \theta_t, \phi_t, x_t, y_t)} \right| < \delta$$

$\sqrt{A_i}$ symbolizes that distance from the center of the irradiated area.

Radiance measurements for BTDF

Case 2

$W \ll \sqrt{A_i}$



If there is not lateral propagation (transparency), only one elementary area dA_i contributes

Blue trajectories symbolize contributions from different parts of A_i which have to be summed up

$$\mathrm{BTDF}(\theta_\mathrm{i}, \phi_\mathrm{i}; \theta_\mathrm{t}, \phi_\mathrm{t}) = \frac{dL_\mathrm{t}}{dE}$$

BTDF can be derived from BSSRDF, however, a spatial resolution by the measurement is no prerequisite

$$BTDF = \int_{A_\mathrm{i}} BSSRDF \, \mathrm{d}A_\mathrm{i}$$

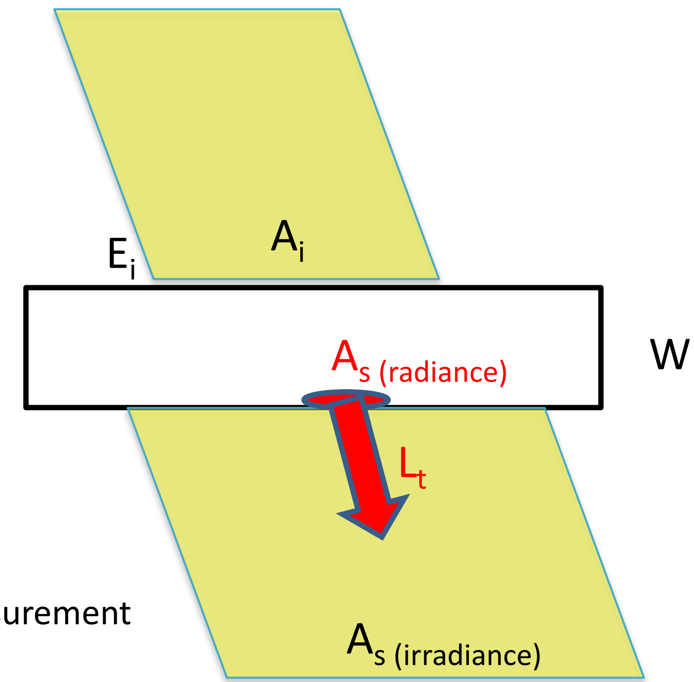
$$\left| 1 - \frac{\int_{A_\mathrm{i}} BSSRDF \, \mathrm{d}A_\mathrm{i}}{\int_{A_\mathrm{i}+A_+} BSSRDF \, \mathrm{d}A_\mathrm{i}} \right| < \delta$$

A_i is enlarged until no further change in L_t is detected within a given uncertainty δ

Irradiance measurements for BTDF

Case 2

W $\ll \sqrt{A_i}$



If there is not lateral propagation (transparency), the size is preserved

Description by a radiance measurement

$$\mathrm{BTDF}(\theta_\mathrm{i},\phi_\mathrm{i};\theta_\mathrm{t},\phi_\mathrm{t}) = \frac{dL_\mathrm{t}}{dE_i}$$

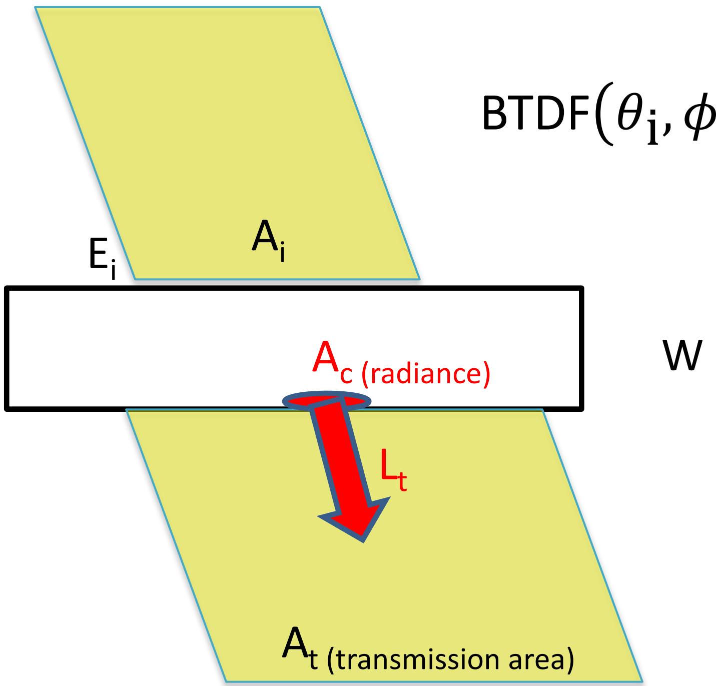
A_i is enlarged until no further change in L_t is detected within a given uncertainty

Description by an irradiance measurement

$$\mathrm{BTDF}(\theta_\mathrm{i},\phi_\mathrm{i};\theta_\mathrm{t},\phi_\mathrm{t}) = \frac{1}{\omega_t \cos\theta_t}\left(\frac{\Phi_t}{\Phi_i}\right)\left(\frac{A_i}{A_s}\right) \overset{*}{=} \frac{1}{\omega_t \cos\theta_t}\left(\frac{\Phi_t}{\Phi_i}\right)$$

$$\left|1 - \frac{A_\mathrm{i}}{A_\mathrm{S} = A_\mathrm{i} + A_+}\right| < \delta$$

Irradiance measurements for BTDF



$$\mathrm{BTDF}(\theta_\mathrm{i},\phi_\mathrm{i};\theta_\mathrm{t},\phi_\mathrm{t}) = \frac{1}{\omega_t \cos\theta_t}\left(\frac{\Phi_t}{\Phi_i}\right)\left(\frac{A_i}{A_t}\right) = \frac{1}{\omega_t \cos\theta_t}\left(\frac{\Phi_r}{\Phi_i}\right)$$

Only if

$$\left|1 - \frac{A_\mathrm{i}}{A_\mathrm{t} = A_\mathrm{i} + A_+}\right| < \delta$$

This must be considered if E_i is not uniform across A_i.

Identity holds in cases where lateral propagation is small.

A relative error $U_{rel,\ irr}$ in irradiance mode may be calculated by $U_{rel,\ irr} = \frac{\Phi_\mathrm{t} - \Phi_\mathrm{t}(A_\mathrm{t} = A_\mathrm{i})}{\Phi_\mathrm{t}}$, if the above equation is applied, where $\Phi_\mathrm{t}(A_t = A_\mathrm{i})$ is the radiant flux transmitted within $A_\mathrm{t} = A_\mathrm{i}$ and Φ_t is the collected radiant flux.

Conclusions

- If the thickness is much larger than the irradiated area, BSSRDF measurement must be done.
- If the thickness is small BTDF measurements can be done.
- With very small lateral propagation, BTDF measurements must be done, in irradiance or in radiance.
- If lateral propagation is high, a radiance measurement must be applied.
- When dealing with structured surfaces, Ai and A_s must be larger than typical structure lengths to achieve sufficient averaging

Thanks for your attention